Availability and Reliability Analysis for the k-out-of-n:G system with Three Failures using Markov Model

M. A. El-Damcese, N. H. El-Sodany

Abstract — This paper presents Markov models for analyzing the reliability and availability for the k-out-of-n:G repairable system with three failures [partial failure, complete failure under repair, complete failure under maintenance]. In this study, we assumed that the working time and the repair time of each component are both arbitrary distributed (e.g., exponentially, Weibull distributed). The Markov method is used to develop generalized expressions for system state probabilities, system availability and system reliability. Numerical results have been facilitated with the help of Runge-Kutta 4th order method to illustrate the performance of the model by the aid of Maple program.

Keywords — k-out-of-n:G system, Markov model, availability, reliability.

1 INTRODUCTION

Reliability has always been a key role in the design of engineering systems. The most frequently used function in lifetime data analysis and reliability engineering is the reliability or survival function. This function gives the probability of an item operating for a certain amount of time without failure. Exponential distribution is the most commonly used in determining the lifetime reliability of a population of components. There is a great interest in evaluating the reliability and availability of k-out-of-n system model.

An n-component system that works (or is good) if and only if at least k of the n components work (or are good) is called a k-out-of-n:G system. An n-component system that fails if and only if at least k of the n components fail is called a k-out-of-n:F system. Based on these two definitions, a k-out-of-n:G system is equivalent to an (n − k + 1)-out-of-n:F system. The term k-out-of-n system is often used to indicate either a G system or an F system or both. Since the value of n is usually larger than the value of k, redundancy is generally built into a k-out-of-n system. There is a great interest in evaluating the reliability of k-out-of-n:G system, mainly because such systems are more general than series or parallel systems.

Several authors have considered k-out-of-n:G systems. Arulmozhi [1] proposed an algorithm for computing the reliability of k-out-of-n:G system. Arulmozhi [2] presented a simple computation method for determining the system reliability of k-out-of-n systems having unequal and equal reliabilities for components. El-Damcese [3] presented continuous-time homogeneous Markov process to evaluate availability, reliability and MTTF for circular consecutive k-out-of-n:G system with repairman. El-Damcese [4] analyzed the k-out-of-n:G system model with critical human errors, common-cause failures and time dependent system repair-rate. Eryilmaz [5] found the distribution and expected value of the number of working components at time t in usual and weighted k-out-of-n:G systems under the condition that they are working at time t. Haggag and Khayar [6] presented a mathematical model of k-out of-n repairable system is studded with standby units involving human and common-cause failure to evaluate availability, steady state availability, MTTF and cost function with and without repair using supplementary variable technique, Laplace transforms of various state probabilities. Jia et al. [7] introduced the copula method to calculate the reliability of dependent consecutive k-out-of-n:G system. The copula is a popular tool for modeling the dependence structure of data. Kumar and Bajaj [8] analyzed the vague reliability of k-out-of-n:G system (particularly, series and parallel system) with independent and non-identically distributed components, where the reliability of the components are unknown. The reliability of each component has been estimated using statistical confidence interval approach. Then we converted these statistical confidence interval into triangular fuzzy numbers. Based on these triangular fuzzy numbers, the reliability of the k-out-of-n:G system has been calculated. Moustafa [9] presented a continuous time Markov chain (CTMC) model to obtain closed form expressions of the mean time between system failures (MTBF) for K-out-of-n:G systems subject to M exponential failure modes and repairs.

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Tang et al. [10] proposed detailed explanations and rules to derive the static unavailability by the micro-Markov models for the k-out-of-n:G systems with multiple failure modes. Zhang et al. [11] analyzed the k-out-of-(M+N):G warm standby system. In the system, not all components in standby can be treated as identical as they have different failure and repair rates.

This paper presents a continuous time Markov chain (CTMC) to obtain a general formula for the reliability and availability of the k-out-of-n:G system of n-identical and independent components subject to three types of failures and the repaired component is good as new. A component can fail either due to a partial failure or complete failure under repair or due to complete failure under maintenance. In this paper we consider that the system and its components has three states: up, degraded, and down. The transition from up state to degraded state represents a partial failure, and the transition from up state to down state or from degraded state to down state represents a complete failure.

The outline of the paper is as follows. The basic assumptions and notations used are given in Section 2. Section 3 is voted to the analysis of k-out-of-n:G system. The availability and reliability of the system are obtained in this section. In Section 4 we give a numerical example of the system when $n=3$ and $k=2$ and the reliability and availability of the system are obtained by the aid of Maple program when the time-to-failure of each component follows exponential distribution and when the time-to-failure of each component follows Weibull distribution. Some concluding remarks are given in Section 5.

2 MODEL DESCRIPTION

We develop the Markov model for n-component system and these components are identical and repairable. At time $t=0$ the system is considered to be in good state and it fails when at least k of the n components fail. The system or the components may fail either due to partial failure or complete failure under repair or complete failure under maintenance. The assumptions and notations, on which the present analysis is based upon, are as follows:

- The system is composed of n-identical and independent components.
- Each component is failed with failure rate $\lambda_i(t), i=1,2,3,4$.
- The failed component is repaired with repair rate $\mu_i(t), i=1,2,3$.
- At time $t=0$ all components are up, and the system can work if and only if at least k of the n components work (or are good).
- The system cannot return to the working condition when $n-k+1$ components completely fail.

Notations:

n : the number of components in the system.

k : the minimum number of components that must work for the k-out-of-n:G system to work.

$\lambda_1(t)$: the failure rate of a component when it goes from up state to degraded state.

$\lambda_2(t)$: the failure rate of a component when it goes from up state to down state under repair.

$\lambda_3(t)$: the failure rate of a component when it goes from up state to down state under maintenance.

$\lambda_4(t)$: the failure rate of a component when it goes from degraded state to down state under repair.

$\mu_1(t)$: the repair rate of a component when it goes from degraded state to up state.

$\mu_2(t)$: the repair rate of a component when it goes from down state under repair to up state.

$\mu_3(t)$: the repair rate of a component when it goes from down state under maintenance to up state.

(m_1,m_2,m_3) : the state of the system, where m_1, m_2 and m_3 represent the number of failed components due to partial failure, complete failure under repair and complete failure under maintenance, respectively.

$P_t(m_1,m_2,m_3)$: the probability that the system is in state (m_1,m_2,m_3) at time t.

3 ANALYSIS OF THE K-OUT-OF-N:G SYSTEM

Now we construct the system availability differential difference equations governing the model as follows:

State Probabilities:

The state probabilities for the system, $P_t(m_1,m_2,m_3)$, $m_1,m_2,m_3=0,1,\dots,n$ can be viewed as a result of solving the following set of first order differential equations:

For $m_1=m_2=m_3=0$:

$$\frac{dP_t(0,0,0)}{dt}=-\left(n\sum_{i=1}^{3}\lambda_i(t)\right)P_t(0,0,0)+\mu_1(t)P_t(1,0,0)+\mu_2(t)P_t(0,1,0)+\mu_3(t)P_t(0,0,1) \quad (1.1)$$

For $1\le m_1\le n$:

$$\frac{dP_t(m_1,0,0)}{dt}=-\left((n-m_1)\sum_{i=1}^{3}\lambda_i(t)+m_1\lambda_4(t)+m_1\mu_1(t)\right)P_t(m_1,0,0)+(n-m_1+1)\lambda_1(t)P_t(m_1-1,0,0)+(m_1+1)\mu_1(t)P_t(m_1+1,0,0)+\mu_2(t)P_t(m_1,1,0)+\mu_3(t)P_t(m_1,0,1) \quad (1.2)$$

For $0 \le m_1 < n$, $0 \le m_2 , m_3 \le n-k$:

$$\frac{dP_t(m_1,m_2,m_3)}{dt} = -P_t(m_1,m_2,m_3)\left(\left(n-\sum_{i=1}^{3} m_i\right)\sum_{i=1}^{3}\lambda_i(t)+m_1\lambda_4(t)+\sum_{i=1}^{3} m_i\mu_i(t)\right)$$
$$+(m_1+1)\mu_1(t)P_t(m_1+1,m_2,m_3)$$
$$+(m_2+1)\mu_2(t)P_t(m_1,m_2+1,m_3)$$
$$+(m_3+1)\mu_3(t)P_t(m_1,m_2,m_3+1) \quad (1.3)$$
$$+\left(n-\sum_{I=1}^{3} m_i+1\right)\Big(\lambda_1(t)P_t(m_1-1,m_2,m_3)$$
$$+\lambda_2(t)P_t(m_1,m_2-1,m_3)+\lambda_3(t)P_t(m_1,m_2,m_3-1)\Big)$$
$$+(m_1+1)\lambda_4(t)P_t(m_1+1,m_2-1,m_3)$$

For $m_2+m_3 = n-k+1, \quad m_1 = 0$:

$$\frac{dP_t(m_1,m_2,m_3)}{dt} = -\left(\sum_{i=1}^{3} m_i\mu_i(t)\right)P_t(m_1,m_2,m_3)$$
$$+\left(n-\sum_{I=1}^{3} m_i+1\right)\Big(\lambda_1(t)P_t(m_1-1,m_2,m_3) \quad (1.4)$$
$$+\lambda_2(t)P_t(m_1,m_2-1,m_3)+\lambda_3(t)P_t(m_1,m_2,m_3-1)\Big)$$
$$+(m_1+1)\lambda_4(t)P_t(m_1+1,m_2-1,m_3)$$

For $m_2+m_3 = n-k+1, \quad m_1 = 1,\ldots,k-1$:

$$\frac{dP_t(m_1,m_2,m_3)}{dt} = -\left(\sum_{i=2}^{3} m_i\mu_i(t)\right)P_t(m_1,m_2,m_3)$$
$$+\left(n-\sum_{I=1}^{3} m_i+1\right)\Big(\lambda_1(t)P_t(m_1-1,m_2,m_3) \quad (1.5)$$
$$+\lambda_2(t)P_t(m_1,m_2-1,m_3)+\lambda_3(t)P_t(m_1,m_2,m_3-1)\Big)$$
$$+(m_1+1)\lambda_4(t)P_t(m_1+1,m_2-1,m_3)$$

Initial Conditions:

At time $t=0$, the state probabilities satisfy the following initial conditions:

$$P_0(0,0,0)=1$$
$$P_0(m_1,m_2,m_3)=0 \text{ for } 1 \le m_1+m_2+m_3 \le n \quad (2)$$

3.1 System Availability

The system availability is the summation of the probabilities of all working states, the general form solution of the k-out-of-n:G system availability at time t is given by:

$$A(t)=\sum_{m=0}^{n} P_t(m_1,m_2,m_3) \quad (3)$$
$$, m = m_1+m_2+m_3 \qquad , m_2+m_3 \ne n-k+1$$

Here, according to the values of k and n we use the numerical method based on the Runge-Kutte 4th order method to find the solution of the system availability $A(t)$ given by (3) with the initial conditions equation (2).

3.2 System Reliability

To obtain the reliability function of the system, we consider that the set of failed states, $m_2+m_3=n-k+1$ are absorbing states.

Now let:

$$P_t(m_1,m_2,m_3) \to \tilde{p}_t(m_1,m_2,m_3)$$

The general form solution of the k-out-of-n:G system reliability at time t is given by:

$$R(t)=\sum_{m=0}^{n} \tilde{P}_t(m_1,m_2,m_3) \qquad , m_2+m_3 \ne n-k+1 \quad (4)$$
$$, m = m_1+m_2+m_3$$

Here, according to the values of k and n we use the numerical method based on the Runge-Kutte 4th order method to find the solution of the system reliability $R(t)$ given by (4) with the initial conditions (2).

4 AN ILLUSTRATIVE EXAMPLE

Consider the k-out-of-n:G system when $k=2$ and $n=3$. Table 1 gives the event space of states of the system.

Table 1: Event space of states of the system

$m = m_1+m_2+m_3$	(m_1,m_2,m_3)
0	$(0,0,0)$
1	$(1,0,0)$, $(0,1,0)$, $(0,0,1)$
2	$(1,1,0)$, $(0,1,1)$, $(1,0,1)$, $(2,0,0)$, $(0,2,0)$, $(0,0,2)$
3	$(1,1,1)$, $(1,2,0)$, $(1,0,2)$, $(2,1,0)$, $(2,0,1)$, $(3,0,0)$

The repair time of each component follows exponential distribution, we can then write:

$$\mu_1(t)=0.10$$
$$\mu_2(t)=0.07$$
$$\mu_3(t)=0.09$$

The time-to-failure distribution of each component is exponential distribution as a particular case, we can then write:

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$\lambda_1(t) = 0.06$

$\lambda_2(t) = 0.02$

$\lambda_3(t) = 0.04$

$\lambda_4(t) = 0.08$

The time-to-failure distribution of each component is Weibull distribution as a particular case, we can then write:

$\lambda_1(t) = 0.06\,t^{0.1}$

$\lambda_2(t) = 0.02\,t^{0.2}$

$\lambda_3(t) = 0.04\,t^{0.3}$

$\lambda_4(t) = 0.08\,t^{0.4}$

For the 2-out-of-3:G system, the working states are $(0,0,0)$, $(1,0,0)$, $(0,1,0)$, $(0,0,1)$, $(2,0,0)$, $(1,1,0)$, $(1,0,1)$, $(3,0,0)$, $(2,1,0)$, $(2,0,1)$, and the failed states are $(0,1,1)$, $(0,0,2)$, $(0,2,0)$, $(1,1,1)$, $(1,2,0)$, $(1,0,2)$.

4.1 2-Out-Of-3:G System Availability

Fig. 1 gives the state transition diagram of the 2-out-of-3:G system.

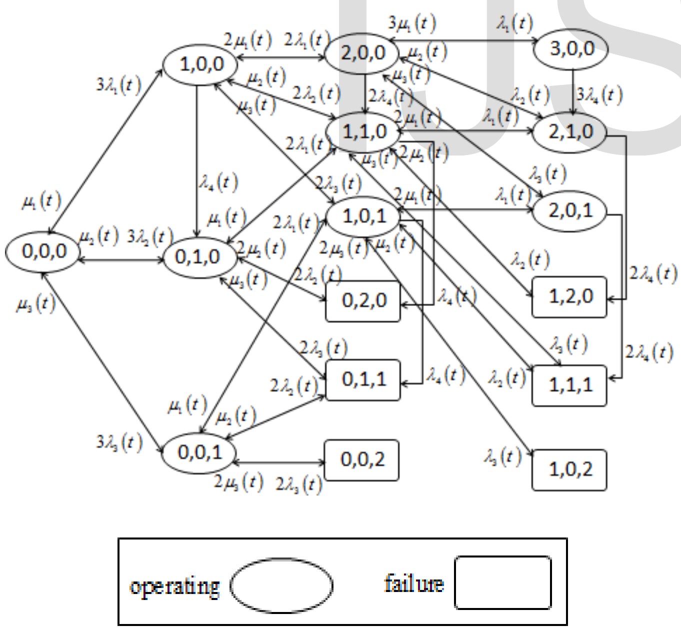

Figure 1: State transition diagram of the 2-out-of-3:G system.

The set of differential equations associated with the system are given by:

$$\frac{dP_t(0,0,0)}{dt} = -3\sum_{i=1}^{3}\lambda_i(t)P_t(0,0,0) + \mu_1(t)P_t(1,0,0) + \mu_2(t)P_t(0,1,0) + \mu_3(t)P_t(0,0,1) \quad (5.1)$$

$$\frac{dP_t(1,0,0)}{dt} = -\left(2\sum_{i=1}^{3}\lambda_i(t) + \lambda_4(t) + \mu_1(t)\right)P_t(1,0,0) + 2\mu_1(t)P_t(2,0,0) + \mu_2(t)P_t(1,1,0) + \mu_3(t)P_t(1,0,1) + 3\lambda_1(t)P_t(0,0,0) \quad (5.2)$$

$$\frac{dP_t(0,1,0)}{dt} = -\left(2\sum_{i=1}^{3}\lambda_i(t) + \mu_2(t)\right)P_t(0,1,0) + \mu_1(t)P_t(1,1,0) + 2\mu_2(t)P_t(0,2,0) + \mu_3(t)P_t(0,1,1) + 3\lambda_2(t)P_t(0,0,0) + \lambda_4(t)P_t(1,0,0) \quad (5.3)$$

$$\frac{dP_t(0,0,1)}{dt} = -\left(2\sum_{i=1}^{3}\lambda_i(t) + \mu_3(t)\right)P_t(0,0,1) + \mu_1(t)P_t(1,0,1) + \mu_2(t)P_t(0,1,1) + 2\mu_3(t)P_t(0,0,2) + 3\lambda_3(t)P_t(0,0,0) \quad (5.4)$$

$$\frac{dP_t(2,0,0)}{dt} = -\left(\sum_{i=1}^{3}\lambda_i(t) + 2\lambda_4(t) + 2\mu_1(t)\right)P_t(2,0,0) + 3\mu_1(t)P_t(3,0,0) + \mu_2(t)P_t(2,1,0) + \mu_3(t)P_t(2,0,1) + 2\lambda_1(t)P_t(1,0,0) \quad (5.5)$$

$$\frac{dP_t(1,1,0)}{dt} = -\left(\sum_{i=1}^{3}\lambda_i(t) + \lambda_4(t) + \mu_1(t) + \mu_2(t)\right)P_t(1,1,0) + 2\mu_1(t)P_t(2,1,0) + 2\mu_2(t)P_t(1,2,0) + \mu_3(t)P_t(1,1,1) + 2\lambda_2(t)P_t(1,0,0) + 2\lambda_4(t)P_t(2,0,0) + 2\lambda_1(t)P_t(0,1,0) \quad (5.6)$$

$$\frac{dP_t(1,0,1)}{dt} = -\left(\sum_{i=1}^{3}\lambda_i(t) + \lambda_4(t) + \mu_1(t) + \mu_3(t)\right)P_t(1,0,1) + 2\mu_1(t)P_t(2,0,1) + \mu_2(t)P_t(1,1,1) + 2\mu_3(t)P_t(1,0,2) + 2\lambda_1(t)P_t(0,0,1) + 2\lambda_3(t)P_t(1,0,0) \quad (5.7)$$

$$\frac{dP_t(0,2,0)}{dt} = -2\mu_2(t)P_t(0,2,0) + 2\lambda_2(t)P_t(0,1,0) + \lambda_4(t)P_t(1,1,0) \quad (5.8)$$

$$\frac{dP_t(0,1,1)}{dt} = -\left(\mu_2(t) + \mu_3(t)\right)P_t(0,1,1) + 2\lambda_2(t)P_t(0,0,1) + 2\lambda_3(t)P_t(0,1,0) + \lambda_4(t)P_t(1,0,1) \quad (5.9)$$

$$\frac{dP_t(0,0,2)}{dt} = -2\mu_3(t)P_t(0,0,2) + 2\lambda_3(t)P_t(0,0,1) \quad (5.10)$$

$$\frac{dP_t(3,0,0)}{dt} = -\left(3\mu_1(t) + 3\lambda_4(t)\right)P_t(3,0,0) + \lambda_1(t)P_t(2,0,0) \quad (5.11)$$

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$$\frac{dP_t(2,1,0)}{dt} = -\left(2\mu_1(t)+\mu_2(t)+2\lambda_4(t)\right)P_t(2,1,0) + \lambda_1(t)P_t(1,1,0)+\lambda_2(t)P_t(2,0,0)+3\lambda_4(t)P_t(3,0,0) \quad (5.12)$$

$$\frac{dP_t(2,0,1)}{dt} = -\left(2\mu_1(t)+\mu_3(t)+2\lambda_4(t)\right)P_t(2,0,1) + \lambda_1(t)P_t(1,0,1)+\lambda_3(t)P_t(2,0,0) \quad (5.13)$$

$$\frac{dP_t(1,2,0)}{dt} = -2\mu_2(t)P_t(1,2,0)+\lambda_2(t)P_t(1,1,0) + 2\lambda_4(t)P_t(2,1,0) \quad (5.14)$$

$$\frac{dP_t(1,1,1)}{dt} = -\left(\mu_2(t)+\mu_3(t)\right)P_t(1,1,1)+\lambda_2(t)P_t(1,0,1) + \lambda_3(t)P_t(1,1,0)+2\lambda_4(t)P_t(2,0,1) \quad (5.15)$$

$$\frac{dP_t(1,0,2)}{dt} = -2\mu_3(t)P_t(1,0,2)+\lambda_3(t)P_t(1,0,1) \quad (5.16)$$

According to (3), the availability function for the 2-out-of -3:G system is given by:

$$A(t) = P_t(0,0,0)+P_t(1,0,0)+P_t(0,1,0)+P_t(0,0,1) + P_t(2,0,0)+P_t(1,1,0)+P_t(1,0,1)+P_t(3,0,0) + P_t(2,1,0)+P_t(2,0,1) \quad (6)$$

At time $t=0$, the state probabilities given by (5.1 to 5.16) satisfy the following initial conditions:

$P_0(0,0,0)=1$

, and all other initial probabilities are equal to zero.

When the time-to-failure of each component follows exponential distribution: using Maple program, the 2-out-of-3:G system availability versus time is shown in Fig .2.

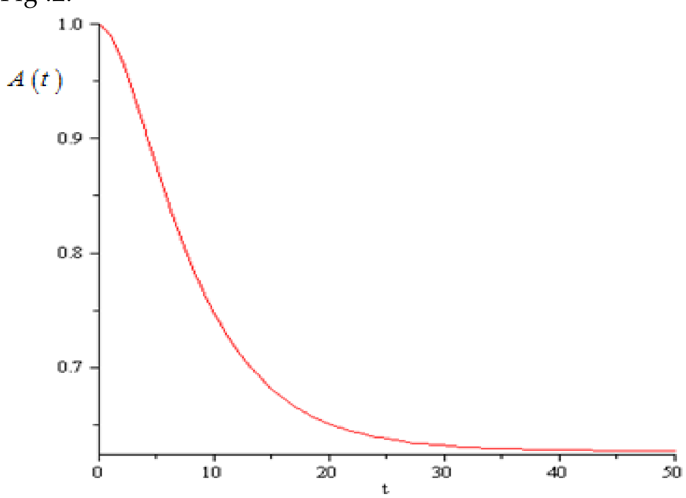

Figure 2: 2-out-of-3:G system availability $A(t)$ versus time t when the time-to-failure of each component follows exponential distribution.

When the time-to-failure of each component follows Weibull distribution: using Maple program, the 2-out-of-3:G system availability versus time is shown in Fig .3.

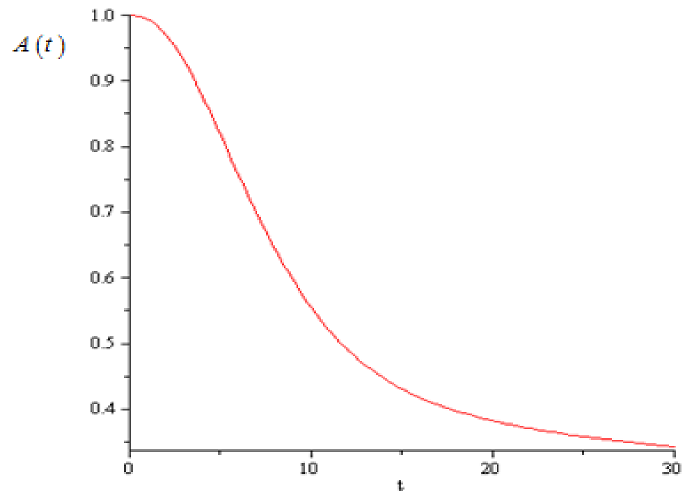

Figure 3: 2-out-of-3:G system availability $A(t)$ versus time t when the time-to-failure of each component follows Weibull distribution.

4.2 2-Out-Of-3:G System Reliability

Fig. 4 gives the state transition diagram of the 2-out-of-3:G system in case the set of failed states, $m_2+m_3=2$ are absorbing states.

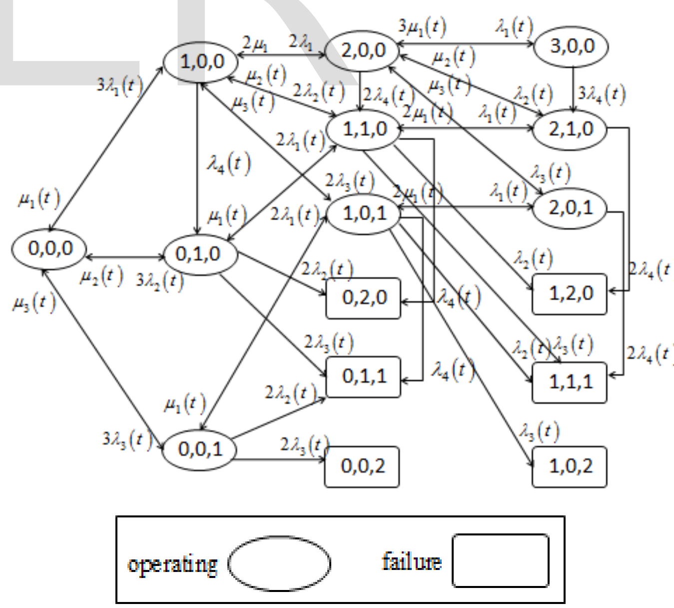

Figure 4: State transition diagram of the 2-out-of-3:G system in case the set of failed states are absorbing states.

To obtain the reliability function of the 2-out-of-3:G system, we consider that the set of failed states, $m_2+m_3=2$ are absorbing states.

The set of differential equations associated with the system are given by:

$$\frac{d\tilde{P}_t(0,0,0)}{dt}=-3\sum_{i=1}^{3}\lambda_i(t)\tilde{P}_t(0,0,0)+\mu_1(t)\tilde{P}_t(1,0,0)+\mu_2(t)\tilde{P}_t(0,1,0)+\mu_3(t)\tilde{P}_t(0,0,1) \quad (7.1)$$

$$\frac{d\tilde{P}_t(1,0,0)}{dt}=-\left(2\sum_{i=1}^{3}\lambda_i(t)+\lambda_4(t)+\mu_1(t)\right)\tilde{P}_t(1,0,0)+2\mu_1(t)\tilde{P}_t(2,0,0)+\mu_2(t)\tilde{P}_t(1,1,0)+\mu_3(t)\tilde{P}_t(1,0,1)+3\lambda_1(t)\tilde{P}_t(0,0,0) \quad (7.2)$$

$$\frac{d\tilde{P}_t(0,1,0)}{dt}=-\left(2\sum_{i=1}^{3}\lambda_i(t)+\mu_2(t)\right)\tilde{P}_t(0,1,0)+\mu_1(t)\tilde{P}_t(1,1,0)+3\lambda_2(t)\tilde{P}_t(0,0,0)+\lambda_4(t)\tilde{P}_t(1,0,0) \quad (7.3)$$

$$\frac{d\tilde{P}_t(0,0,1)}{dt}=-\left(2\sum_{i=1}^{3}\lambda_i(t)+\mu_3(t)\right)\tilde{P}_t(0,0,1)+\mu_1(t)\tilde{P}_t(1,0,1)+3\lambda_3(t)\tilde{P}_t(0,0,0) \quad (7.4)$$

$$\frac{d\tilde{P}_t(2,0,0)}{dt}=-\left(\sum_{i=1}^{3}\lambda_i(t)+2\lambda_4(t)+2\mu_1(t)\right)\tilde{P}_t(2,0,0)+3\mu_1(t)\tilde{P}_t(3,0,0)+\mu_2(t)\tilde{P}_t(2,1,0)+\mu_3(t)\tilde{P}_t(2,0,1)+2\lambda_1(t)\tilde{P}_t(1,0,0) \quad (7.5)$$

$$\frac{d\tilde{P}_t(1,1,0)}{dt}=-\left(\sum_{i=1}^{3}\lambda_i(t)+\lambda_4(t)+\mu_1(t)+\mu_2(t)\right)\tilde{P}_t(1,1,0)+2\mu_1(t)\tilde{P}_t(2,1,0)+2\lambda_2(t)\tilde{P}_t(1,0,0)+2\lambda_4(t)\tilde{P}_t(2,0,0)+2\lambda_1(t)\tilde{P}_t(0,1,0) \quad (7.6)$$

$$\frac{d\tilde{P}_t(1,0,1)}{dt}=-\left(\sum_{i=1}^{3}\lambda_i(t)+\lambda_4(t)+\mu_1(t)+\mu_3(t)\right)\tilde{P}_t(1,0,1)+2\mu_1(t)\tilde{P}_t(2,0,1)+2\lambda_1(t)\tilde{P}_t(0,0,1)+2\lambda_3(t)\tilde{P}_t(1,0,0) \quad (7.7)$$

$$\frac{d\tilde{P}_t(0,2,0)}{dt}=2\lambda_2(t)\tilde{P}_t(0,1,0)+\lambda_4(t)\tilde{P}_t(1,1,0) \quad (7.8)$$

$$\frac{d\tilde{P}_t(0,1,1)}{dt}=2\lambda_2(t)\tilde{P}_t(0,0,1)+2\lambda_3(t)\tilde{P}_t(0,1,0)+\lambda_4(t)\tilde{P}_t(1,0,1) \quad (7.9)$$

$$\frac{d\tilde{P}_t(0,0,2)}{dt}=2\lambda_3(t)\tilde{P}_t(0,0,1) \quad (7.10)$$

$$\frac{d\tilde{P}_t(3,0,0)}{dt}=-\left(3\mu_1(t)+3\lambda_4(t)\right)\tilde{P}_t(3,0,0)+\lambda_1(t)\tilde{P}_t(2,0,0) \quad (7.11)$$

$$\frac{d\tilde{P}_t(2,1,0)}{dt}=-\left(2\mu_1(t)+\mu_2(t)+2\lambda_4(t)\right)\tilde{P}_t(2,1,0)+\lambda_1(t)\tilde{P}_t(1,1,0)+\lambda_2(t)\tilde{P}_t(2,0,0)+3\lambda_4(t)\tilde{P}_t(3,0,0) \quad (7.12)$$

$$\frac{d\tilde{P}_t(2,0,1)}{dt}=-\left(2\mu_1(t)+\mu_3(t)+2\lambda_4(t)\right)\tilde{P}_t(2,0,1)+\lambda_1(t)\tilde{P}_t(1,0,1)+\lambda_3(t)\tilde{P}_t(2,0,0) \quad (7.13)$$

$$\frac{d\tilde{P}_t(1,2,0)}{dt}=\lambda_2(t)\tilde{P}_t(1,1,0)+2\lambda_4(t)\tilde{P}_t(2,1,0) \quad (7.14)$$

$$\frac{d\tilde{P}_t(1,1,1)}{dt}=\lambda_2(t)\tilde{P}_t(1,0,1)+\lambda_3(t)\tilde{P}_t(1,1,0)+2\lambda_4(t)\tilde{P}_t(2,0,1) \quad (7.15)$$

$$\frac{d\tilde{P}_t(1,0,2)}{dt}=\lambda_3(t)\tilde{P}_t(1,0,1) \quad (7.16)$$

According to (4), the reliability function for the 2–out–of –3:G system is given by:

$$R(t)=\tilde{P}_t(0,0,0)+\tilde{P}_t(1,0,0)+\tilde{P}_t(0,1,0)+\tilde{P}_t(0,0,1)+\tilde{P}_t(2,0,0)+\tilde{P}_t(1,1,0)+\tilde{P}_t(1,0,1)+\tilde{P}_t(3,0,0)+\tilde{P}_t(2,1,0)+\tilde{P}_t(2,0,1) \quad (8)$$

At time $t=0$, the state probabilities given by (7.1 to 7.16) satisfy the following initial conditions:

$\tilde{P}_0(0,0,0)=1$

, and all other initial probabilities are equal to zero.

When the time-to-failure of each component follows exponential distribution: using Maple program, the 2-out-of-3:G system reliability versus time is shown in Fig .5.

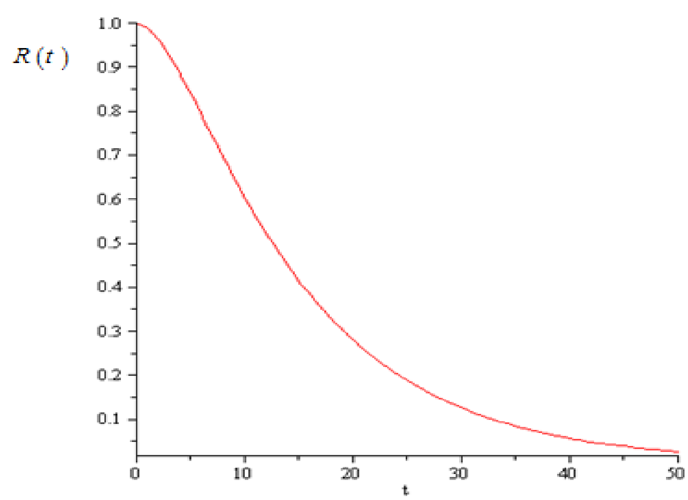

Figure 5: 2-out-of-3:G system reliability $R(t)$ versus time t when the time-to-failure of each component follows exponential distribution.

When the time-to-failure of each component follows Weibull distribution: using Maple program, the 2-out-of-3:G system reliability versus time is shown in Fig .6.

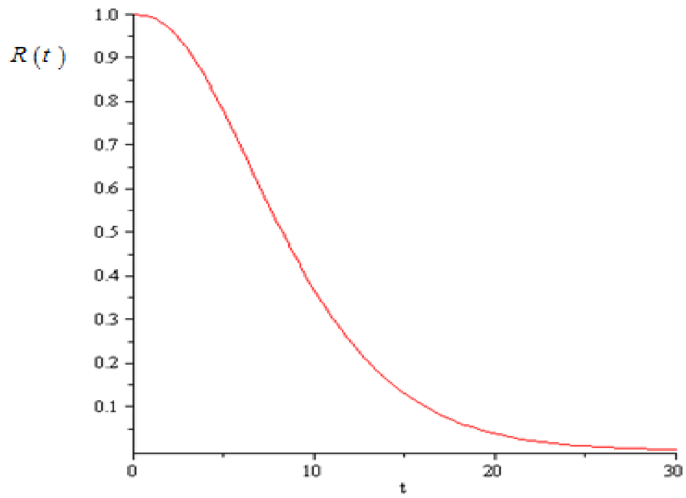

Figure 6: 2-out-of-3:G system reliability $R(t)$ versus time t when the time-to-failure of each component follows Weibull distribution.

5 CONCLUSION

In this paper, a mathematical model was constructed for the k-out-of-n:G system subject to three types of failures, partial failure, complete failure under repair and complete failure under maintenance. The system and its components has three states: up, degraded, and down. Availability and reliability were obtained for a system consists of three components and the results were shown graphically by the aid of MAPLE program when the time-to-failure of each component follows arbitrary distributions. The results obtained in this paper can be applied to similar models.

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